Energy deposition of heavy ions in the regime of strong beam-plasma correlations

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The energy loss of highly charged ions in dense plasmas is investigated. The applied model includes strong beam-plasma correlation via a quantum *T*-matrix treatment of the cross sections. Dynamic screening effects are modeled by using a Debye-like potential with a velocity dependent screening length that guarantees the known low and high beam velocity limits. It is shown that this phenomenological model is in good agreement with simulation data up to very high beam-plasma coupling. An analysis of the stopping process shows considerably longer ranges and a less localized energy deposition if strong coupling is treated properly.

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One of the frequently discussed scenarios for inertial confinement fusion is the indirectly heavy ion-driven one [1]. Here, the intense ion beams heat a converter which in turn provides radiation that uniformly compresses the fusion capsule. In the first stage, the converter is turned into a plasma with solid state density and temperatures from a few to hundreds of eV. Fast heavy ions are rapidly ionized to high charge states in this environment.

Due to these conditions, strong beam-plasma correlations occur in converters. Assuming that the screening length of the target electrons $\lambda_D^e=(k_BT_e/4\pi e^2 n_e)^{1/2}$ is a measure of the interaction length, the coupling strength can be described for slow beam ions by

$$\gamma(v=0)=\frac{\langle E_{pot}\rangle}{\langle E_{kin}\rangle}\approx\frac{Z_b e^2}{\lambda_D^e k_B T_e}\approx Z_b\Gamma_{ee}^{3/2}. \qquad (1)$$

Here, $\Gamma_{ee}=e^2(4\pi n_e/3)^{1/3}/k_BT_e$ is the classical nonideality parameter of the plasma electrons, and Z_b is the beam charge number. For higher beam velocities v, the kinetic energy can be estimated by a smooth interpolation between the thermal and relative beam energies, i.e., $\langle E_{kin}\rangle\approx[(k_BT_2)^2+(m_e v^2)^2]^{1/2}$. Of course, screening becomes less effective for fast beam particles. If we use an effective screening length $\lambda=\lambda_D^e(1+v^2/v_{th}^2)^{1/2}$ to model this effect (see below), the parameter γ takes the form

$$\gamma(v)\approx Z_b\Gamma_{ee}^{3/2}\left(1+\frac{v^2}{v_{th}^2}\right)^{-3/2}, \qquad (2)$$

with $v_{th}=(k_BT_e/m_e)^{1/2}$ being the electron thermal velocity. As expected, the beam-plasma coupling strength decreases for faster beam ions.

To model the energy transfer for the situation of interest, the stopping power theory has to cover the regime from the strongly coupled low velocity case to the weakly coupled high beam energy limit, where collective effects are important. In particular, both strong correlations and collective phenomena have to be included at intermediate beam velocities.

The case of weak beam-plasma correlations can be described, e.g., by the dielectric formalism [2], which gives the well-known Bethe-like result for $v\to\infty$,

$$\lim_{v\to\infty}\frac{\partial}{\partial x}\langle E\rangle=-\frac{Z_b^2e^2\omega_{pl}^2}{v^2}\ln\left(\frac{2m_ev^2}{\hbar\omega_{pl}}\right). \qquad (3)$$

$\omega_{pl}=(4\pi e^2n_e/m_e)^{1/2}$ is the electron plasma frequency. There exist several attempts to extent the weak coupling theories into the regime of strong beam-plasma correlations. Z_b^3 corrections were discussed [3,4] and terms beyond the Fokker-Planck approach were considered [5]. In a more general approach, the energy loss was expressed in terms of the force autocorrelation function [6]. Furthermore, phase shift calculations using a self-consistent effective potential determined by a density functional technique were done [8,7]. On the other hand, computer simulations were performed to obtain the stopping power up to large coupling parameters [9,10].

To include strong beam-plasma correlations in quantum kinetic theory, a *T*-matrix approach with a dynamically screened Born term in random phase approximation (RPA) and statically screened higher-order ladder contributions was applied in Refs. [11,12]. In this way, both collective excitations (plasmons) and strong binary correlations were accounted for approximately. Important features of this approach are the correct asymptotics (3) and the good agreement with particle in cell (PIC) and molecular dynamics (MD) simulations for $v\to 0$ and coupling strengths up to $Z_b\Gamma^{3/2}=10$ [12,13]. Reasonable agreement with numerical simulation data could also be obtained for intermediate beam velocities and moderate coupling strength ($Z_b\Gamma^{3/2}<0.3$) [12].

However, increasing deviations occur for larger $Z_b\Gamma^{3/2}$. Here, we want to recall that dynamic screening is included only in the first Born term on the RPA level if the combined scheme of Refs. [11,12] is applied. However, dynamic screening is expected to become significant in all ladder diagrams for moderate beam velocities and strong coupling, i.e., a full dynamically screened ladder approximation (see, e.g., Ref. [14]) is required here. Moreover, a more rigorous many-body approach should also include nonlinear screening beyond the RPA approximation. This is an extremely difficult problem, and up to now a rigorous solution is not visible.

A quite phenomenological and simple model to include dynamic screening effects in higher-order ladder terms is based on a modified Debye potential

$$V_{ab}^{S}(r,v)=\frac{e_a e_b}{r}\exp[-r/\lambda(v)], \tag{4}$$

where the dynamics of screening is modeled by a velocity dependent screening length $\lambda(v)$. Now the idea is to adjust the effective screening length in such a way that the correct asymptotic results for the stopping power are obtained. This can be done via [10]

$$\lambda(v)=\lambda_D^e\left(1+\frac{v^2}{v_{th}^2}\right)^{1/2}. \tag{5}$$

Debye screening, i.e., $\lambda=\lambda_D^e=v_{th}/\omega_{pl}$, is obtained in the limit $v\to 0$, while $\lambda(v)=v/\omega_{pl}$ follows for high beam velocities.

In the following, we will use the effective potential (4) with the screening length (5) to calculate the stopping power at the *T*-matrix level. Starting from the quantum Boltzmann equation, we obtain for the free carrier contributions to the stopping power of a nondegenerate plasma [12,15]

$$\frac{\partial}{\partial x}\langle E\rangle=-\sum_c \frac{m_c^2}{\mu_{bc}^3}\frac{n_c\Lambda_c^3}{(2\pi)^2\hbar^3}\frac{k_BT}{v}\int_0^\infty dp\,p^3 Q_{bc}^T(p,\lambda(v)) \times\left\{p_-\exp\left(-\frac{m_c v_-^2}{2k_BT}\right)-p_+\exp\left(-\frac{m_c v_+^2}{2k_BT}\right)\right\}. \tag{6}$$

Here, we introduced $\mu_{bc}=m_b m_c/(m_b+m_c)$ (reduced mass), $\Lambda_c=[2\pi\hbar^2/(m_c k_B T)]^{1/2}$ (thermal wave length), $p_\pm=1\pm(\mu_b k_B T)/(m_c p v)$ and $v_\pm=p/\mu_b\pm v$. The sum in Eq. (6) runs over all carrier species in the plasma, but except for very slow beam particles only the free electrons have to be considered.

The transport cross section Q_{bc}^T is the central quantity in Eq. (6). In the considered nondegenerate case, it is appropriate to calculate the cross section via

$$Q_{bc}^T(p,\lambda(v))=\frac{4\pi\hbar^2}{p^2}\sum_{l=0}^{\infty}(l+1)\sin^2(\eta_l-\eta_{l+1}). \tag{7}$$

The scattering phase shifts η_l are obtained by numerical solution of the radial Schrödinger equation which corresponds to the determination of the full *T* matrix. Since we will use potential (4), dynamic screening effects are modeled in all ladder terms of the *T* matrix.

In Fig. 1, the change of the cross section with beam velocity is demonstrated. In the limit $v\to 0$, we have static Debye screening. In general, we observe increasing cross sections at higher beam velocities due to weaker screening. Resonance states appear as sharp peaks in the low energy range. They follow from the contributions of former bound state energy levels which are merged into the scattering continuum under the influence of the surrounding medium. The Born approximation, which is exemplary shown for $\lambda(7v_{th})$, is sufficient to determine the stopping power at high beam velocities.

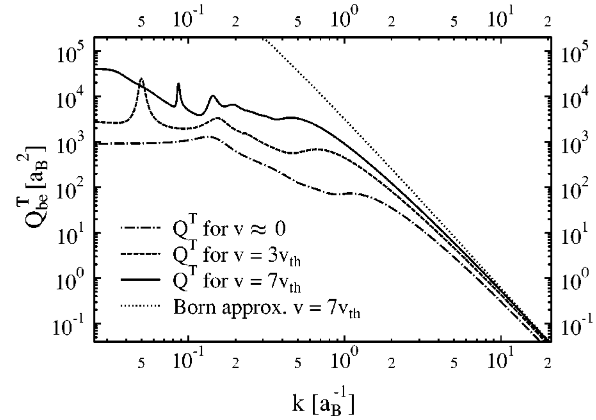

FIG. 1. Transport cross section vs wave number $k=p/\hbar$ for different screening lengths λ which are related to given beam velocities by Eq. (5) [$\lambda(0)=\lambda_D=1.41\,a_B$].

Results for the stopping power for an ion beam moving in an electron gas are shown in Fig. 2. The beam ion charge number Z_b is here assumed to be constant. The considered beam and plasma parameters result in large beam-plasma coupling parameters. The considered approaches for the stopping power give quite different results. As expected the dynamically screened Born approximation given by [12,15]

$$\frac{\partial}{\partial x}\langle E\rangle=\frac{2Z_b^2e^2}{\pi v^2}\int_0^\infty\frac{dk}{k}\int_{(\hbar k^2/2m_b)-kv}^{(\hbar k^2/2m_b)+kv} \times d\omega\left[\omega-\frac{\hbar k^2}{2m_b}\right]\mathrm{Im}\,\varepsilon_{\mathrm{RPA}}^{-1}(k,\omega)n_{\mathrm{B}}(\omega) \tag{8}$$

overestimates the stopping power considerably (dashed line). The inclusion of strong binary collisions on the *T*-matrix level reduces the stopping power. Evidently, the modeling of the dynamics of screening in the *T*-matrix calculation using potential (4) with (5) (solid line) leads to a larger reduction of the stopping power than the combined *T*-matrix scheme of Refs. [11,12] (dash-dotted line). Moreover, the velocity dependent screening model strongly improves the agreement with the simulation data in the regime of large beam-plasma coupling (similar results follow if the velocity dependent screening length in the *T*-matrix approach is determined by fitting the statically screened Born approximation to the dynamic RPA results of the stopping power instead of using Eq. (5) [16]). The deviations between the different models increase with the coupling strength. Further investigations show that the *T*-matrix model with the effective screening length (5) coincides with the combined scheme of Ref. [12] for weak and moderate beam-plasma coupling ($Z_b\Gamma_{ee}^{3/2}<0.3$); agreement with the Born approximation is only achieved for very hot plasmas ($T_e>10^7$ K).

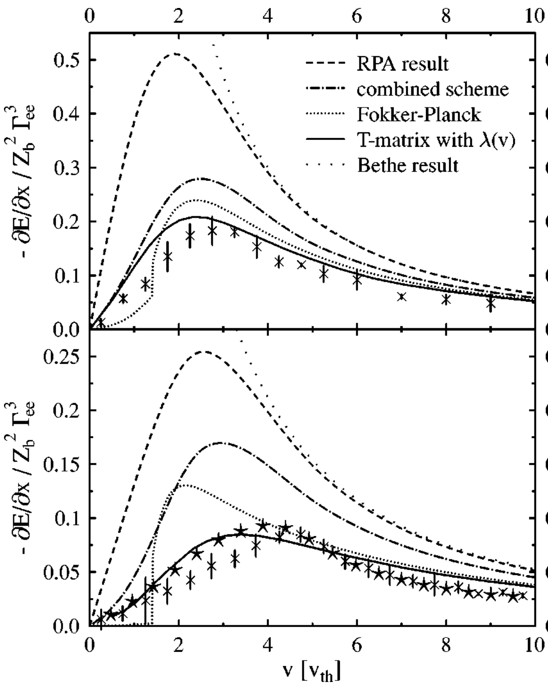

FIG. 2. Comparison of different approaches for the stopping power (see text) with data from MD (crosses) and PIC (stars) simulations. The beam charge number is $Z_b = 10$. The target is an electron gas with $n_e = 1.25\times10^{22}\,\mathrm{cm}^{-3}$, $T_e = 1.84\times10^5$ K ($Z_b\Gamma_{ee}^{3/2} = 1.98$) in the upper panel and $n_e = 9.9\times10^{22}\,\mathrm{cm}^{-3}$, $T_e = 1.15\times10^5$ K ($Z_b\Gamma_{ee}^{3/2} = 11.22$) in the lower panel. The stopping power is given in units of $3(k_BT)^2/e^2$.

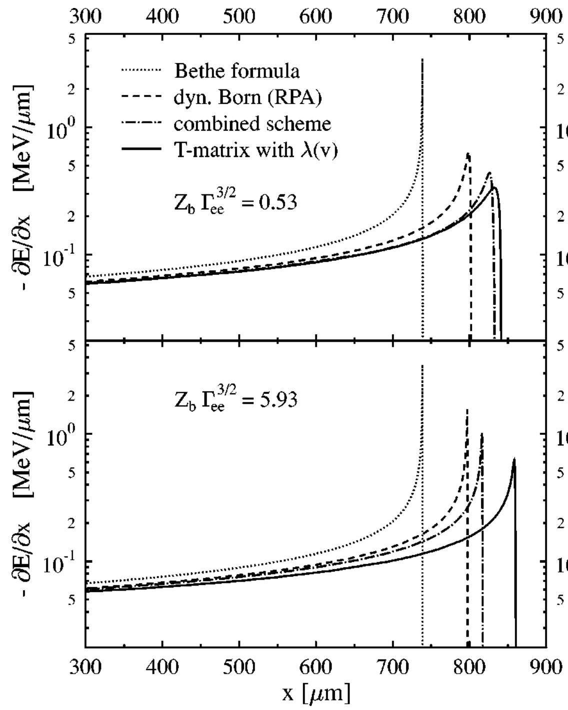

FIG. 3. Energy deposition vs distance traveled by the beam ion in the plasma considering different approximations for the stopping power. The beam particle is a $^{12}C^{+6}$ ion with an initial beam energy of $E = 6$ MeV per nucleon. The target is an electron gas with $n_e = 5\times10^{22}\,\mathrm{cm}^{-3}$ and $T_e = 5\times10^5$ K ($T_e = 1\times10^5$ K) in the upper (lower) figure.

For comparison, results of an extended Fokker-Planck approach [5,17] are also shown. In this approach, collective excitations were artificially included by an additional term which vanishes for $v < \sqrt{2}v_{th}$, and otherwise enhances the results in a way that the Bethe-like asymptotics (3) follows for $v \to \infty$. In the regime of strong coupling, this leads to a large jump and to a stopping power which is too small for $v < v_{th}$. Nevertheless, this approach works well for $v > 4v_{th}$, where the coupling parameter $\gamma(v)$ is not too large.

Although the presented *T*-matrix model with a velocity dependent screening length is not based on a rigorous quantum many-body theory, it provides the possibility to treat strong collisions including dynamic screening effects with a standard approach for the cross sections and to obtain the correct high velocity result (3) without free parameters. Furthermore, we found that the model agrees also well with simulation data for $v \to 0$ [12,13] and intermediate beam velocities (see Fig. 2) up to high coupling strengths.

It turns out that in the classical regime, the *T*-matrix results depend only on the parameter $Z_b\Gamma_{ee}^{3/2}$. This behavior has been exploited to construct a fit formula for the stopping power that covers a wide range of beam and plasma parameters [13]. Furthermore, we found an increasing reduction of the usual Z_b^2 scaling with increasing coupling strength if the *T*-matrix models are applied.

Now we will investigate the influence of strong correlations on the slowing down process of a beam ion. The time evolution of the beam particle energy and position is determined by the set of equations

$$\dot{x} = v(t) \quad \text{and} \quad \dot{v} = \frac{1}{m_b}\frac{\partial}{\partial x}\langle E(v)\rangle, \tag{9}$$

with the initial conditions $v(0) = v_0 = [2E(v_0)/m_b]^{1/2}$ and $x(0) = 0$. The quantity x denotes the distance a beam ion has traveled in the plasma.

We will first consider the stopping of light beam ions where the ions can assumed to be fully ionized except for the low velocities at the end of the stopping range. Figure 3 shows results for the energy loss versus penetration depth for fully ionized carbon ions with initial beam velocities of $v_0 \approx 12\,v_{th}$ (upper part) and $v_0 \approx 28\,v_{th}$ (lower part). For such initial conditions all approximations show the same well-known qualitative behavior: the energy transfer is first slowly increasing and then sharply peaked close to the point where the particle is stopped (Bragg peak).

Due to the smaller stopping force, the *T*-matrix approaches show a larger penetration depth than the RPA approach and, especially, than the calculations based on the Bethe formula. Compared to the total stopping range, considerable differences of the Bragg-peak positions occur. Furthermore, the Bragg peak is more pronounced in the Bethe and RPA description.

One would expect that strong coupling effects are less important for higher plasma temperatures. However, for intermediate initial beam velocities as considered in Fig. 3 this is not the case. The reason is that the ratio v/v_{th} is lower now, and therefore, lower velocities, where strong beam-plasma coupling occurs, are relevant for a larger fraction of the stopping range. Only for very hot plasmas, the results of the RPA and *T*-matrix schemes merge.

Finally, we will consider the case of heavy ion stopping. Here, the charge of the ions has to be determined self-consistently as a function of the beam velocity. Rigorously, this requires a many-body approach describing the kinetics of electron capture and loss processes in the plasma medium. For simplification, we will use here the semiempirical Betz formula [19]

$$Z_b = Z_c \left\{ 1 - \exp\left[-0.555 \left(\frac{m_e v^2}{27.2 eV} Z_c^{-0.607} \right) \right] \right\}, \qquad (10)$$

which is in good agreement with experimental data for partially ionized plasmas [20] (Z_c is the atomic number). Neglecting the contribution of bound plasma electrons, results for the energy deposition of a tungsten beam are shown in Fig. 4. The velocity dependence of the charge number shifts the maximum of the stopping power to higher beam velocities. Accordingly, the Bragg peak is broadened except for the results using Bethe formula (3). Again, the *T*-matrix approach predicts larger penetration depths and a more delocalized energy deposition than the RPA and, in particular, the Bethe results.

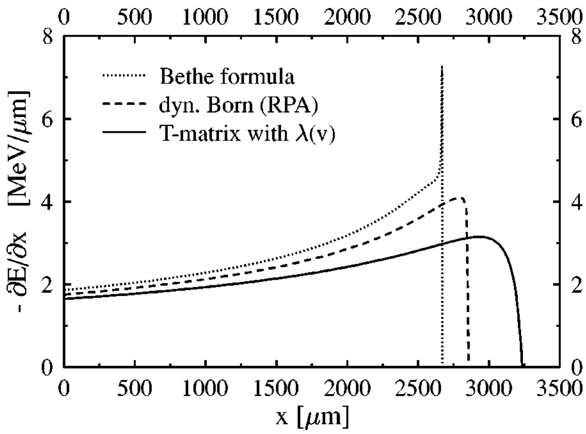

FIG. 4. Energy deposition vs distance traveled by the ion for a beam of $^{209}Bi^{Z+}$ ions with initially $E=35$ MeV per nucleon. The charge state Z_b of the beam ions was calculated with the Betz formula [19]. The target plasma is an electron gas with $n_e=5\times10^{22}$ cm^{-3} and $T_e=5\times10^5$ K.

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